ANSWERS TO THE ODD-NUMBERED PROBLEMS

Chapter1, page 5

1. 10.

3. 1.

5. 1 2 1
   1 2 1
   1 3 3 1.

7. 1, 1 + 5 = 6, 5 + 10 = 15, 10 + 10 = 20, 10 + 5 = 15, 5 + 1 = 6, and 1;
   1, 1 + 6 = 7, 6 + 15 = 21, 15 + 20 = 35, 20 + 15 = 35, 15 + 6 = 21, 6 + 1 = 7, and 1.

9. $\binom{9}{7} = \binom{9}{2} = 36$, $\binom{9}{8} = \binom{9}{1} = 9$.

11. $125x^3 + 150x^2y + 60xy^2 + 8y^3$.

17. $c = 56$, $m = 5$.

19. (a) 4. (b) 8. (c) 16. (d) 32. (e) 64. (f) 128.

21. (a) 0. (b) 0.

23. (a) 8. (b) 16. (c) 32. (d) 64.

25. $r = 6$, $s = 7$, $t = 6$, $u = 7$.

27. $\binom{996}{990}$.

31. $x^4 + 4x^3y + 6x^2y^2 + 4xy^3 + y^4 + 4x^3z + 12x^2yz + 12xy^2z +$
    $4y^3z + 6x^2z^2 + 12xyz^2 + 6y^2z^2 + 4xz^3 + 4yz^3 + z^4$.

33. 3.

37. 3.

39. 36, of which 27 are odd.

41. 0, 1, 3, 7, and 15.

Chapter 2, page 14

3. 103.

7. $F_{n+4} = 3F_{n+1} + 2F_n$.

9. 321.

13. 702.

19. (a) $F_{16} - 1$. (b) $F_{19} - 1$.

21. (a) 2. (b) 3. (c) 5. (d) 8. (e) 13. (f) 21.

23. $r = 4$, $s = 5$, and $t = 11$.

25. $r = 15$, $s = 17$, and $t = 19$.

29. 204

33. $L_{2m+2} - 2$.

35. $r = 4$, $s = 6$, and $t = 8$.

37. (a) 1. (b) -1. (c) 1. (d) -1.

Chapter 3, page 18

1. (a) 5040. (b) 36. (c) 362,880. (d) 720.

5. (a) 5!. (b) 7!. (c) $(n + 1)!$.

7. $a = 14$ and $b = 10$ or $a = 24024$ and $b = 24023$.

9. $n^4 + 10n^3 + 35n^2 + 50n + 24$.

11. (a) 23. (b) 119. (c) 719.

15. $a = 6$, $b = 11$, and $c = 5$.

17. $(2m + 2)! - 1$.

19. (a) $(n + 2)! + (n + 1)! - 2$.

Chapter 4, page 23

1. (a) -4, 3, 10. (b) -19, -23, -27.

3. (a) 990. (b) 993. (c) 988.

5. 33/2.

7. (a) -33,698. (b) 501,994. (c) -11,385/4. (d) n^2.

9. 54, 1458, 13122.

11. 686, 4802, 33614.

13. ± 42.

15. (a) $(7^{1000} - 1)/6$. (b) $(1 - 7^{1000})/8$. (c) $(7^n - 1)/6$.

17. 9.

21. (a) $6\sqrt{3}$. (b) $6\sqrt{3}$. (c) 4. (d) 4.

23. 30.

27. $(x^{n+1} - y^{n+1})/(x - y)$.

29. (a) 5. (b) 24 miles per hour.

31. $2[1 - (1/2)^7]$.

35. (a) 0, 3, 5, 6, 9, 10, 12, 15, 18, 20, 21, 24, 25, 27, 30.
(b) 15. (c) 15. (d) $16 \bullet 15 = 240$.

Chapter 5, page 35

13. (a) $1 - \dfrac{1}{n+1}$. (b) $\dfrac{x^{2^{n+1}} - y^{2^{n+1}}}{x - y}$.

15. $(n + 1)/2n$ for $n > 1$.

21. Using mathematical induction, one can show that:

(a) $F_{n+1}^2 - F_n F_{n+2} = (-1)^n$.

(b) $\dfrac{F_{n+2}^2 - F_{n+1}^2}{F_n} = F_{n+3}$.

(c) $F_{n-1} + F_{n+1} = L_n$.

Chapter 6, page 47

1. (a) 4,845. (b) 3,003. (c) -2,912.

3. $a = -3$, $b = 2$, $c = 0$.

7. 1, -1, 1, -1, 1, -1.

11. $\binom{-m}{r} = (-1)^r \binom{r+m-1}{m-1}$.

15. $(d/2)n^2 + [a - (d/2)]n$.

19. $\left(\sum_{i=1}^{2} a_i\right)\left(\sum_{i=1}^{2} b_i\right) = a_1b_1 + a_1b_2 + a_2b_1 + a_2b_2$ and $\sum_{i=1}^{2} (a_ib_i) = a_1b_1 + a_2b_2$. These are not always equal, since, for example, they are unequal for $a_1 = a_2 = b_1 = b_2 = 1$.

23. $(1/6)n^3 - (1/2)n^2 + (1/3)n$.

25. $n^3 + 5n$.

27. $s = 3$, $t = 1$.

29. $r = 6$, $s = 7$, $t = 1$.

31. $(1/5)n^5 + (1/2)n^4 + (1/3)n^3 - (1/30)n$.

37. $(n^3 + 3n^2 + 2n)/6$.

Chapter 7, page 55

1. *a, b; a, c; a, d; a, e; b, c; b, d; b, e; c, d; c, e; d, e.*

3. *a, b; b, a; a, c; c, a; a, d; d, a; a, e; e, a; b, c; c, b; b, d; d, b; b, e; e, b; c, d; d, c; c, e; e, c; d, e; e, d.*

5. 7,920.

7. 30.

9. $\binom{12}{3}\binom{9}{7}$, $\binom{12}{3}\binom{9}{2}$, $\binom{12}{7}\binom{5}{3}$, $\binom{12}{7}\binom{5}{2}$, $\binom{12}{2}\binom{10}{7}$, $\binom{12}{2}\binom{10}{3}$.

11. -1,201,200.

17. 3^{100}.

19. 1,2,3,4; 2,1,4,3; 3,1,2,4; 4,1,3,2;
1,3,4,2; 2,4,3,1; 3,2,4,1; 4,3,2,1;
1,4,2,3; 2,3,1,4; 3,4,1,2; 4,2,1,3.

23. (a) 200. (b) 10,150.

Chapter 8

Sections 8.1, 8.2 and 8.3, page 64

1. $(x + 2)(x^3 + 3x^2 - 6x + 2) - 16$.

3. $-1 \pm \sqrt{3}$.

5. $a = -27{,}604$.

7. $-1,\ 3,\ \sqrt{2}i,\ -\sqrt{2}i$.

9. $f(0) = 0 = f(a)$. Two factors are x and $x - a$.

11. 0, -2, -4, -2/3.

13. $\frac{5}{3}, \frac{-3+\sqrt{17}}{2}, \frac{-3-\sqrt{17}}{2}$.

15. $0, -\frac{1}{3}, \frac{2}{3}, \frac{1+\sqrt{3}i}{6}, \frac{1-\sqrt{3}i}{6}$.

17. ±1, ±2, ±3, ±5, ±6, ±10, ±15, ±30.

25. $x^8 - 48x^6 + 536x^4 - 1728x^2 + 400 = 0$.

27. (a) b.
(b) $c(2x + 1) + b$.
(c) $d(3x^2 + 3x + 1) + c(2x + 1) + b$.
(d) $\binom{n}{1}x^{n-1} + \binom{n}{2}x^{n-2} + \binom{n}{3}x^{n-3} + \cdots + \binom{n}{n-1}x + \binom{n}{n}$.

29. (a) 0. (b) 0. (c) $6d$.

33. $a = 38$, $b = 84$, $c = 81$, $d = 34$, $e = 5$.

39. No.

Section 8.4, page 68

1. (a) 4. (b) 8/3. (c) 16. (d) 32/3. (e) 16/3. (f) $\frac{4\sqrt{3}}{3}$.

5. 1.

7. $a = p^3 - 2pq$, $b = p^4q - 3p^2q^2 + 2q^3$, $c = p^3q^3 - 2pq^4$, $d = q^6$.

9. (a) 0. (b) -3. (c) 1. (d) $y^3 - 3y - 1 = 0$.

11. (a) $x = 3$, $y = -3$, $z = 1$.
(b) $t = 4$, $u = 2$, $v = -4$, $w = 1$.

Chapter 9

Sections 9.1 and 9.2, page 78

1. $x = -6/53$, $y = -91/53$.

3. $x = 103/21$, $y = -71/21$, $z = -4/63$.

5. $x = 5$, $y = 2$, $z = 4$.

7. $x = (b + c - a)/2$, $y = (c + a - b)/2$, $z = (a + b - c)/2$.

Sections 9.3, page 85

15. $4xyz$.

17. (b) $x = -1$, $y = 2$, $z = 0$.

    (c) $x = 2$, $y = -3$, $z = 3$, $w = 0$.

    (d) $$\begin{vmatrix} L_{n+2} & L_{n+1} \\ L_{n+1} & L_n \end{vmatrix} = -\begin{vmatrix} L_{n+1} & L_n \\ L_n & L_{n-1} \end{vmatrix},$$

    $$L_{2n} = \frac{3L_n^2 - 2L_nL_{n+1} + 2L_{n+1}^2}{5},$$

    $$L_{3n} = \frac{2L_n^3 - 3L_n^2L_{n+1} + 3L_nL_{n+1}^2}{5}.$$

Section 9.4, page 90

5. $(w - x)(w - y)(w - z)(z - x)(z - y)(y - x)$.

7. $(w - x)(w - y)(w - z)(z - x)(z - y)(y - x)$.

9. $r = 2$, $s = 2$, $t = 1$.

11. $r = 4$, $s = 2$, $t = 2$.

13. (b) $x = 1$, $y = 1$, $z = 1$.

Chapter 10

Section 10.1, page 95

19. $a = 2$, $b = 3$, $c = 4$.

Sections 10.3 and 10.4, page 103

21. (a) 101/100.

23. (a) The arithmetic mean is 2; the geometric mean is $\sqrt[4]{2}$.
    (b) $a = 15$, $b = -20$, $c = 15$, $d = -6$.
    (c) The roots are equal, and so all equal to 1, since their arithmetic mean is the same as their geometric mean.

INDEX

A

Absolute value of a real number, 68, 89, 94
Advice to the problem solver, vii
Analogue of a formula, 16-17, 37, 38, 50
Angles of a polygon, 35
Arithmetic mean, 22, 25-28, 99-101, 103, 106, 107
Arithmetic progression, 21-26, 34
Arrangements, 53-55, 57
Average (*see* Arithmetic mean)
Axiomatic approach, 29

B

Binomial, 3
Binomial coefficients, 2-11, 15-17, 25, 34, 37, 40-45, 47-53, 56, 106
Binomial expansion (*see* Binomial coefficients)
Binomial Theorem, 41-43

C

Cauchy, Augustin (1789-1857), 102
Cauchy-Buniakowski Inequality, 102-108
Cauchy-Schwarz Inequality, 102-108
Closure, 94
Coefficients:
- binomial, 2-11, 15-17, 25, 34, 37, 40-45, 47-53, 56, 106
- complex, 58, 68
- integral, 63-66
- of linear equations, 71-73
- multinomial, 53-56
- of a polynomial, 58, 61, 63-66, 68-70
- trinomial, 53-56

Cofactor, 77, 79, 82, 85-86
Combinations, 51-57
Complex coefficients, 58, 68
Complex numbers, 1, 58, 59, 68
Consecutive integers, 33, 38
Consecutive terms of a progression, 22-25
Constructible lengths, 1
Convex polygons, 29, 35
Cramer's Rule, 73, 76, 78, 84-85
Crout's Method, 85
Cube roots, 1, 23

D

Deduction, 29
Degree of a polynomial, 58, 59, 64, 67
Determinant, 71-93
Determinant of the system, 73, 75, 84
Determinants of large order, 81-93
Diagonal:
- of the Pascal Triangle, 3-4, 40
- of a polygon, 29-31
- of a square, 1

Difference:
- of an arithmetic progression, 21-23
- of a function, 66-67

Discriminant of a quadratic, 97-99
Division, synthetic, 60-63, 67
Divisor, integral, 63-65
Double subscript notation, 76, 81

E

Elimination technique, 71-73, 75, 85
Equations:
- linear, 71-92
- polynomial, 38, 58-70, 98-99, 107

Even permutation, 54-57, 76, 77, 82, 83, 84

F

Factor Theorem, 59, 89
Factorials, 18-20, 34, 41-43, 46, 48, 51-54, 82
Factorizations, 26, 32-33, 36, 59, 65-66, 68, 89-93
Fibonacci (c. 1170-1250), 12
Fibonacci numbers, 12-17, 27, 34, 37-38, 50, 88, 99, 108
Fibonacci Quarterly, The, 14
Fibonacci sequence (*see* Fibonacci numbers)
Final value in a summation, 46
Finite sequence, 21, 22
First-degree equation, 71
Fixed multiplier, 22
Fractions, 1
Function, 58, 66-70, 97-99, 102
Fundamental Theorem of Algebra, 68

G

General polynomial, 46, 58
Geometric mean, 23, 25, 26, 99-101, 103, 106, 107
Geometric progression, 22-27, 34

H

Harmonic mean, 101, 106
Harmonic progression, 102

I

Identity, 44
Index of a summation, 45
Induction, 29-39, 41-43, 49, 66, 95, 97, 100, 106
Inequalities, 94-108
Initial value in a summation, 46
Integers, 1, 12, 13, 21, 27, 29-39, 41-49, 56, 57, 63-66, 95-97, 106, 107
Integral coefficients, 63-66
Integral divisor, 63-65
Integral multiple, 27, 33, 36, 39, 49
Integral roots, 63-65
Irrational roots, 64, 66

L

Leonardo of Pisa (c. 1170-1250), 12
Liber Abacci, 12
Linear equations, 71-92
Lucas, Francois Edouard Anatole (1842-1891), 14
Lucas numbers, 14-17, 34-35, 37, 38, 50, 88, 108
Lucas sequence(*see* Lucas numbers)

M

Mathematical induction, (See Induction)
Mean:
- arithmetic, 22, 25-28, 99-101, 103, 106, 107
- geometric, 23, 25, 26, 99-101, 103, 106, 107
- harmonic, 101, 106

Minor of a determinant, 77, 79, 82, 85-86, 89
Multinomial coefficients, 53-56
Multiple, integral, 27, 33, 36, 39, 49
Multiple roots, 67-68
Multiplier, fixed, 22

N

Natural numbers, 1
*n*th roots, 1, 23, 94, 100
Number systems, 1
Numbers:
complex, 1, 58, 59, 68
Fibonacci, (See Fibonacci numbers)
Lucas, (See Lucas numbers)
natural, 1
negative real, 1, 97-98, 99, 103
rational, 1, 64-65
real, 1, 23, 58, 94-108
triangular, 22, 31
Numerical approximation of a determinant, 85

O

Odd permutation, 54-57, 76, 77, 82, 83
Order of a determinant, 72, 75, 81, 85

P

Pascal, Blaise (1623-1662), 3, 29
Pascal Triangle (*see* Binomial coefficients)
Pell sequence, 38
Permutations, 53-57, 76, 77, 82-84, 105
Pi notation, 46, 48, 49, 107
Polygons, 29-31, 35
Polynomial equations, 38, 58-70, 98-99, 107
Polynomials, 19, 46, 48, 49, 58-70, 86, 90, 97-99, 102
Product notation, 46, 48, 49, 107
Progression:
arithmetic, 21-26, 34
geometric, 22-27, 34
harmonic, 102
Proper subset, 55

Q

Quadratic equation, 38, 98-99
Quadratic function, 97-99
Quotient, 22, 60, 63

R

R (before problem numbers), vii
Ratio of a geometric progression, 22-26
Rational numbers, 1, 64-65
Rational roots, 63-65
Real numbers, 1, 23, 58, 94-108
Remainder in a division, 60-63
Remainder Theorem, 59
Roots:
cube, 23
integral, 63-65
irrational, 64, 66
multiple, 67-68
*n*th, 1, 23, 94, 100
of a polynomial equation, 38, 59-69, 98-99
rational, 63-65
square, 23, 97, 98, 99, 106, 108
Row number of a binomial coefficient, 3-4

S

Schwarz, Hermann Amandus (1843-1921), 102
Schwarz Inequality, 102-108
Sequence:
arithmetic (progression), 21-26, 34
factorial, 18
Fibonacci, (See Fibonacci numbers)
geometric (progression), 22-27, 34
Lucas, (See Lucas numbers)
miscellaneous, 28, 38, 50
Pell, 38
Set, 27, 31, 55
Sigma notation, 45-50, 67, 102-103
Simultaneous linear equations, 71-92
Square roots, 23, 97, 98, 99, 106, 108
Strong mathematical induction, 34-35
Subsets, 51, 55
Sum:
of an arithmetic progression, 21, 24
of a geometric progression, 22-26
of squares of binomial coefficients, 10
of squares of real numbers, 102-103
Summation notation, 45-50, 67, 102-103
Symmetric function, 68-70
Symmetry of the Pascal Triangle, 43, 52
Synthetic division, 60-63, 67
System of linear equations (*see* simultaneous linear equations)
Systems, number, 1

T

Theorems, deduction of, 29
Transpose of a determinant, 83, 85
Transpose Theorem, 83, 85
Triangle, sides of a, 97, 108
Triangular numbers, 22, 31
Trichotomy, 94
Trinomial coefficients, 53-56

V

Vandermonde, Alexandre-Theophile (1735-1796), 89
Vandermonde determinants, 89-93